

1 Expected Value

1.1 Concepts

Distribution	PMF	$E(X)$
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	$\sum_{i=1}^n \frac{x_i}{n}$
Bernoulli Trial	$f(0) = 1 - p, f(1) = p$	p
Binomial	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$	np
Geometric	$f(k) = (1-p)^k p$	$\frac{1-p}{p}$
Hyper-Geometric	$f(k) = \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}$	$\frac{nm}{N}$
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	λ

The **Expected Value** is the weighted average of all the values the random variables can take on. By definition, it is:

$$E[X] = \sum x_i f(x_i).$$

It satisfies some properties:

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$ for **all** random variables
- $E[XY] = E[X]E[Y]$ for **independent** random variables.

1.2 Examples

2. I flip a fair coin 5 times. What is the expected number of heads I flip?
3. I roll two fair 6 sided die. What is the expected value of their product?
4. I randomly rearrange 10 people of different heights in a line. Let X be the number of people in the right sorted order (person i is the i th tallest person). What is $E[X]$?

1.3 Problems

5. True False The expected value of a random variable X is the value such that the PMF at that point is the largest.

6. True False The expected value of a random variable X always exists.
7. True False We have that $E[X^2] = E[X \cdot X] = E[X]E[X]$.
8. True False If $a \leq X \leq b$ (a is the smallest X can be and b is the largest), then $a \leq E[X] \leq b$.
9. While pulling out of a box of cookies, what is the expected number of cookies I have to pull out before I pull out an oatmeal raisin if 20% of cookies are oatmeal raisin?
10. What is the expected number of aces I have when I draw 5 cards out of a deck?
11. I am rolling two die and I stop when I roll snake eyes (2 1's). What is the expected number of times I have to roll the die?
12. The number of lightning strikes during a thunderstorm is given by a Poisson distribution with expected value 10. What is the probability that there are 5 strikes in the latest storm?
13. In a safari, safari-keepers have caught and tagged 300 rhinos. On a safari, out of the 15 different rhinos you see, there are 5 of them expected to be tagged. How many rhinos are there at the safari?
14. (Challenge) In a class of 30 students, I split them up into 6 groups of 5 on Tuesday. Today, Thursday, I split them up again randomly. What is the expected number of people in your new group were in your old group on Tuesday?